CHAPTER 21 Electric Charge

Answers to Understanding the Concepts Questions

1. The only point where the net force on a test charge is zero is the midpoint between the two charges.

2. Yes, as Kepler's laws require that two objects attract each other with an inverse-square force, be it gravitational or electrostatic. Kepler's first law still holds true, namely, the two charges will move in an elliptical orbit around the center-of-mass of the system (provided that their total energy is negative — see the concept of electrostatic potential energy discussed in Chapter 24) . Since the force is aligned with the line connecting the two charges it provides no torque on either charge, so the angular momentum of each charge is still conserved and Kepler's second law (the law of area) remains valid. Finally, Kepler's third law, a direct consequence of the inverse-square nature of the force, is also true here: T^2 is proportional to r^3.

3. The balloon acquires a charge when rubbed on a sweater. It then induces an opposite charge on the wall through polarization, and the attraction between the charges on the balloon and those on the wall keeps the balloon there for a while.

4. The spark is caused by a transfer of the charge carried by the walker to the door knob. This can happen only if there is a build-up of charge on the walker. In the winter the air is drier; that is, there is less water vapor in the air. Since water molecules are efficient at picking up and carrying off charges on an object in their presence, it is more difficult for you to build up a significant charge in humid air. In the winter there is a better chance for a charge build-up, and therefore of a dramatic discharge.

5. In a short time interval Δt the objects cannot move very far, so we can consider their separation, r, essentially constant. Therefore, the electrostatic repulsion between them is also a constant: $F = kq_1q_2/r^2 \approx$ constant. The distance each can travel follows from $\Delta x_i = \frac{1}{2}a_i(\Delta t)^2 = \frac{1}{2}(F/m_i)t^2 \approx \frac{1}{2}(kq_1q_2/r^2m_i)(\Delta t)^2$, where $i = 1, 2$. So the factors that determine the distance each object can travel are q_1, q_2, r, Δt, and the mass the object in question. The ratio is $\Delta x_1 / \Delta x_2 = m_2 / m_1 = 3$, i.e., the object with three times the mass travels 1/3 of the distance of the other one.

6. The basic principle of the quantitative operation of the electroscope is outlined in Problem 21-5. The measurement of the angle made by a gold leaf can be translated into a measurement of the charge. In order to measure accurately the charge you carry you might want to stand on an insulating mat as you touch the metal top of the electroscope. That both controls the situation and ensures that your charge doesn't leak off elsewhere just as you are trying to measure it.

7. The ones farther from the nucleus are more involved in chemical reactions. This is because chemical reactions involve rearrangement of electronic orbitals, and electrons in an outer orbital are not as strongly tied to the nucleus so they are more likely to be able to rearrange themselves in a chemical reaction.

8. Let's call the quarks with charge $2e/3$ u-quarks, and those with charge $-e/3$ d-quarks (this is the standard nomenclature). Then the following compose all the possible combinations of three quarks, together with their charges: *uuu:* $2e$; *uud:* e; *udd:* 0; *ddd:* $-e$. The second combination has charge

corresponding to that of the proton, while the third combination has charge corresponding to that of the neutron. The other two combinations do not occur as stable particles.

9. Rubbing between the fingers and the packaging material results in charge transfer between the two, causing both to be oppositely charged. The opposite charges then attract each other, and for the small pieces of packaging materials (peanuts) with very little weight, this attraction is relatively strong enough to cause them to stick to the fingers. They are difficult to shake off as their masses are so small, and so the electrostatic attraction is often enough to withstand the violent acceleration of the shaking.

10. As an object is exposed to the air, it can get a fresh supply of electrons from humid air and neutralize the excess positive charge.

11. The net force on an object at the center of the circle is indeed zero. However, this point is not a stable equilibrium point. If the positive charge moves away from the center along the axis of the circular charge distribution, all the forces act to repel it, so that it will accelerate away from the center. The center is thus a point of unstable equilibrium, analogous to a ball resting on the top of a hill. The slightest displacement will cause it to move away from its starting point.

12. As charges move, a certain amount of negative may leave one contact but an equal amount of negative charge would enter the other one, thereby preserving the total amount of charges.

13. Let ball 1 have an initial charge of –4.8 (in units of 10^{-19} C); balls 2,3,4 are initially uncharged. If we assume that the balls are identical, then touching 1 and 2 will give each one of them a charge of –2.4 units, while balls 3 and 4 remain uncharged. If now ball 1 (or 2) is made to touch both 3 and 4 simultaneously, then the three balls each get one third of the available charge, that is –0.8 units.

14. Assuming that the electrostatic force between the two objects is not negligible compared with the their weights. You can suspend each object with a string and place them close together to see whether they move closer to, or away from, each other. The absolute sign of the charge on each object cannot be determined without further information. All we know can determine is whether their signs are the same or opposite.

15. A spark occurs as a result of the charge transfer between your hand and the car door. The fact that the rubber tires are good insulators only means that any excessive charges carried by the car cannot flow into the ground immediately, and that does not prevent the charge transfer from happening between the hand and the metal door knob, which is a conductor.

16. If the electrical charge of a fundamental particle such as the electron depended on its velocity, then we would have a chance to measure the tiny parameter κ only because of a departure from neutrality. Gravity is so weak that for all practical purposes two electrically neutral blocks of material do not exert forces on one another. Under the hypothesis, such objects would only be neutral because any surplus of charge due to a different motion of the electrons and the positive ions would be neutralized by ambient charges. However, we could put the two blocks in a vacuum. If at that point there is no force between the objects, then presumably the values of κ for the electrons and ions are such that at the given temperature the net charge of each object is zero. That is because a given temperature for the electrons corresponds, on average, to a certain value of v_e^2 for the electrons and, by equipartition, another value of v_i^2 for the ions determined by the relation $m_e v_e^2 = m_i v_i^2$. But now all we have to do is to keep the objects in the vacuum and raise the temperature. Each object will now acquire the same net charge, different from zero, and the repulsion should be detectable. We know that any such parameter κ must be very tiny, if it is not zero, because the existence of a temperature-dependent inverse square force has not been observed to the accuracy of our instruments.

17. No. The measurement of e may yield v, but v is only the velocity of the charged particle relative to a certain reference frame, not the absolute velocity, which is meaningless and impossible to measure.

18. No, because the data only show that for electrons and for protons the charge of each does not change. It is entirely possible that in electron-proton collisions at high temperatures (high energies) at some stage of the development of the quasar, some new net charge is produced from a neutral environment. This would correspond to charge nonconservation. As long as the amount of new charge is small, and as long as the processes producing these new charges do not affect the processes which cause the radiation that we observe, charge conservation would not be observed by the study of the colors of the quasar light.

19. If Earth and the Moon each has an equal number of protons and electrons and only the electronic charge is modified, then yes, in principle, Earth and the Moon would each carry a net charge and there would be a, net electrostatic force of the form $1/r^2$ between them. Whether that force would overpower the gravitational force depends on the amount of net charge on each of them. However, it would be more likely for each of them to be electrically neutral when they were formed, meaning that they would each end up with different number of protons and electrons.

20. This is exactly analogous to the motion of a mass inside Earth. We found in Chapter 12 that a mass inside Earth undergoes harmonic motion about the center. A point charge of one sign embedded in a spherically symmetric charge distribution of the opposite sign win also oscillate about the center of the sphere. The frequency can be found when the force inside the sphere can be calculated. This will be done in Chapter 23.

21. The force exerted on q_1 by q_2 doubles, as does that on q_3 by q_2.

22. The electrostatic force is largely responsible for the structures of atoms and molecules making up the star. These particles emit electromagnetic waves, both visible and invisible, than may be detected by us. If the electrostatic force over there were to have a different form than that on Earth, then the wavelengths of the electromagnetic waves emitted from that star would differ significantly from the corresponding values on Earth. This is not the case.

Solutions to Problems

1. The number of electron charge units in the excess positive charge is

   $N = Q/e = (1 \times 10^{-9}\text{ C})/(1.602 \times 10^{-19}\text{ C/electrons}) = 6.2 \times 10^{9}$ electrons,

   so the cork ball has $\boxed{6.2 \times 10^{9}\text{ fewer electrons}}$.

2. Because the uranium atom was initially neutral, the charge is positive:

   $q = +21|e| = +21(1.602 \times 10^{-19}\text{ C}) = \boxed{+3.36 \times 10^{-18}\text{ C}}$.

   The nucleus contains the positive charge of 92 protons:

   $Q = +92(1.602 \times 10^{-19}\text{ C}) = \boxed{1.47 \times 10^{-17}\text{ C}}$.

3. Each molecule of CO_2 contains 6 + 2(8) = 22 electrons. The charge in 1 g is

   $q = [(1\text{ g})/(44\text{ g/mol})](6.02 \times 10^{23}\text{ molecules/mol})(22\text{ electrons/molecule})(1.602 \times 10^{-19}\text{ C/electron})$
   $= \boxed{4.82 \times 10^{4}\text{ C}}$.

4. Because the spheres are identical, charge will be distributed equally when they are connected. After the initial connection, each sphere will have a charge

   $q = \frac{1}{3}Q$.

   The grounded sphere will lose its charge. When it is connected to the other sphere, the charge on that sphere will divide equally:

   $q' = \frac{1}{2}(\frac{1}{3}Q) = Q/6$. Thus the charges will be $\boxed{Q/3, Q/6, Q/6}$.

5. Each gold atom has 79 electrons, so removing one electron in 10^{13} means

   $79 \times 10^{-13} = \boxed{7.9 \times 10^{-12}}$ electrons per atom removed.

6. There are 79 protons per gold atom, and the number of gold atoms in the coin is $N = (28.4\text{ g})/(197\text{ g/mol})(6.022 \times 10^{23}/\text{mol}) = 8.682 \times 10^{22}$, so the total number of protons in the coin is

   $(79\text{ protons/atom})(8.682 \times 10^{22}\text{ atoms}) = \boxed{6.86 \times 10^{24}\text{ protons}}$.

7. From symmetry considerations, each time two identical cork balls touch, the charge is shared evenly. At the first touch, the first cork ball (and the second cork ball) will have $\frac{1}{2}$ of the original charge:

   $q_1 = \frac{1}{2}q_0 = \frac{1}{2}(-4 \times 10^{-10}\text{ C}) = \boxed{-2 \times 10^{-10}\text{ C}}$, and the number of electrons gained is

   $N_1 = (2 \times 10^{-10}\text{ C})/(1.602 \times 10^{-19}\text{ C/electron}) = \boxed{1.25 \times 10^{9}\text{ electrons}}$.

   At the second touch, the second cork ball (and the third cork ball) will have $\frac{1}{2}q_1$:

   $q_2 = \frac{1}{2}q_1 = \frac{1}{2}(-2 \times 10^{-10}\text{ C}) = \boxed{-1 \times 10^{-10}\text{ C}}$, and the number of electrons gained is

   $N_2 = (1 \times 10^{-10}\text{ C})/(1.602 \times 10^{-19}\text{ C/electron}) = \boxed{6.2 \times 10^{8}\text{ electrons}}$.

   $q_3 = q_2 = \boxed{-1 \times 10^{-10}\text{ C},\ 6.2 \times 10^{8}\text{ electrons}}$.

8. From symmetry considerations, each time two identical cork balls touch, the charge is shared evenly. If we touch the first cork ball with an uncharged cork ball, the first cork ball (and the second cork ball) will have 1/2 of the original charge. If we now touch the second cork ball with an uncharged cork ball, the second cork ball and the third cork ball will have 1/4 of the original charge. If we now touch the third cork ball with the last uncharged cork ball, the third cork ball and the fourth cork ball will have 1/8 of the original charge, which is

   $q = (1/8)q_0 = (1/8)(-1.04 \times 10^{-13}\text{ C}) = -0.13 \times 10^{-13}$ C, as desired.

   If we discharge one of the cork balls after touching, we need only one extra cork ball.

$\boxed{9}$. The total number of electrons in 0.1 g of aluminum is

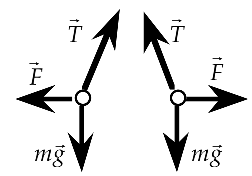
$N = [(0.1\text{ g})/(27.0\text{ g/mol})](6.02 \times 10^{23}\text{ atoms/mol})(13\text{ electron/atom})$

$= 2.9 \times 10^{22}$ electrons.

The fractional increase in the number of electrons is

fraction $= (1 \times 10^{-6}\text{ C})/(1.602 \times 10^{-19}\text{ C/electron})]/(2.9 \times 10^{22}\text{ electrons}) = \boxed{2.2 \times 10^{-10}}$.

10. (a)



(b) Before the charge is added, the cork balls are hanging vertically, so the tension is

$T_1 = mg = (0.2 \times 10^{-3}\text{ kg})(9.8\text{ m/s}^2) = \boxed{2.0 \times 10^{-3}\text{ N}}$.

After the charge is added, the charge will be shared equally by the two cork balls, and there is a horizontal Coulomb force.

From the force diagram, we apply $\sum \vec{F} = 0$:

horizontal: $T \sin\theta = F = kqq/r^2$;

vertical: $T \cos\theta = mg$.

If we divide the two equations, we get

$\tan\theta = F/mg = kq^2/r^2mg = kq^2/(2L\sin\theta)^2mg$

$= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1 \times 10^{-7}\text{ C})^2/[2(0.20\text{ m})\sin\theta]^2(0.2 \times 10^{-3}\text{ kg})(9.8\text{ m/s}^2) = 0.0065/(\sin^2\theta)$.

This equation has only one unknown, θ, but the presence of trigonometric functions makes the algebra a little messy. When we calculate both sides for a range of angles, we get

$\sin\theta = 0.19,\quad \theta = 11°$.

The tension is

$T = mg/(\cos\theta) = (0.2 \times 10^{-3}\text{ kg})(9.8\text{ m/s}^2)/(\cos 11°) = \boxed{2.0 \times 10^{-3}\text{ N}}$.

(c) From the analysis in part (b), we have $\theta = \boxed{11°}$.

11. (a) Because a silicon atom has 14 electrons, we find the number of electrons from

$N = [(5.98 \times 10^{27}\text{ g})/(28\text{ g/mol})](6.02 \times 10^{23}\text{ atoms/mol})(14\text{ electrons/atom})$

$= \boxed{1.8 \times 10^{51}\text{ electrons}}$.

(b) We find the fractional change from

$\Delta q/q = (1 \times 10^{-6}\text{ C})/(1.8 \times 10^{51}\text{ electrons})(1.6 \times 10^{-19}\text{ C/electron}) = \boxed{3.5 \times 10^{-39}}$.

12. Because charge is conserved, the two positive charges on the left must be balanced by two positive charges on the right. The charge of particle X is the proton charge:

$q_X = \boxed{+1.6 \times 10^{-19}\text{ C}}$.

13. (a) For the reaction $p + \bar{p} \rightarrow e^+ + e^- + e^+ + e^- + 2n$, the charges (as multiples of e) are

$+1 - 1 = +1 - 1 + 1 - 1 + 0$. Thus, we have $0 = 0$, so charge is $\boxed{\text{conserved}}$.

(b) For the reaction $e^+ + e^- \rightarrow 2p + n + 2\gamma$, the charges (as multiples of e) are

$+1 - 1 = +2$. Thus, we have $0 \neq 2$, so charge is $\boxed{\text{not conserved}}$.

(c) For the reaction $e^+ + e^- \rightarrow e^+ + e^- + p + \bar{p} + 2\gamma$, the charges (as multiples of e) are

$+1 - 1 = +1 - 1 + 1 - 1 + 0$. Thus, we have $0 = 0$, so charge is $\boxed{\text{conserved}}$.

(d) For the reaction $n + p \rightarrow e^- + p + \bar{p}$, the charges (as multiples of e) are

$0 + 1 = -1 + 1 - 1$. Thus, we have $1 \neq -1$, so charge is $\boxed{\text{not conserved}}$.

14. We let units help us find the charge:

$q = [(6.5 \times 10^{-4}\text{ g})/(9.11 \times 10^{-28}\text{ g/electron})](1.60 \times 10^{-19}\text{ C/electron})$

$= \boxed{1.1 \times 10^5\text{ C}}$.

15. The proton at rest has charge $+ e_0$. The electron has charge $- e_0(1 + v^2/c^2)$, so the net charge is

$$\begin{aligned} Q_{\text{net}} &= + e_0 - e_0(1 + v^2/c^2) \\ &= (1.60 \times 10^{-19}\ \text{C})\{1 - [1 + (1/137)^2]\} \\ &= \boxed{-8.5 \times 10^{-24}\ \text{C}}. \end{aligned}$$

16. For the Coulomb force to be equal to the weight, we have

$k\,e^2/r^2 = mg;$

$(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})^2/r^2 = (1.67 \times 10^{-27}\ \text{kg})(9.8\ \text{m/s}^2)$, which gives

$r = 1.2 \times 10^{-1}\ \text{m} = \boxed{12\ \text{cm}}.$

17. The two up quarks will repel each other with a force

$$\begin{aligned} F_{\text{up-up}} &= kq_1q_2/r_{12}^2 \\ &= (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})(1.6 \times 10^{-19}\ \text{C})/(1.5 \times 10^{-15}\ \text{m})^2 \\ &= \boxed{46\ \text{N repulsion}}. \end{aligned}$$

The up and down quarks will attract each other with a force

$$\begin{aligned} F_{\text{up-down}} &= kq_1q_3/r_{13}^2 \\ &= (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})(1.6 \times 10^{-19}\ \text{C})/(1.5 \times 10^{-15}\ \text{m})^2 \\ &= \boxed{23\ \text{N attraction}}. \end{aligned}$$

18. We equate the two forces:

$F = kq_1q_2/r^2 = mg;$

$r = (kq_1q_2/mg)^{1/2} = [(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.5 \times 10^{-9}\ \text{C})^2/(0.016\ \text{kg} \times 9.8\ \text{m/s}^2)]^{1/2} = \boxed{2.0 \times 10^{-3}\ \text{m}}.$

19. The two ions will repel each other. The magnitude of the Coulomb force is

$F = kq_1q_2/r^2;$

$(1.1 \times 10^{-11}\ \text{N}) = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)q^2/(4.5 \times 10^{-9}\ \text{m})^2$, which gives $q = \boxed{1.6 \times 10^{-19}\ \text{C}}.$

We find the number of electron charges from

$N = q/e = (1.6 \times 10^{-19}\ \text{C})/(1.6 \times 10^{-19}\ \text{C/electron}) = \boxed{1\ \text{electron}}.$

20. We assume that the cork balls are small, so they can be treated as point charges. For the Coulomb force, we have

$F = kq_1q_2/r^2;$

$0.18\ \text{N} = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)q^2/(2 \times 10^{-2}\ \text{m})^2$, which gives

$q = \boxed{8.9 \times 10^{-8}\ \text{C}}.$

If the pith balls were not small, the force between the charges would move some charge to the opposite sides of the pith balls. The center of the charge would not be at the center of the pith balls.

21. Both forces are inverse-square forces, so we have

$$\begin{aligned} F_E/F_g &= (kq^2/r^2)/(Gm^2/r^2) = kq^2/Gm^2 \\ &= (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})^2/(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.10 \times 10^{-3}\ \text{kg})^2 \\ &= \boxed{3.5 \times 10^{-10}}. \end{aligned}$$

This result is so different from Example 21-6 because the masses are so much larger than they are at the atomic scale.

22. For the Coulomb force to be 0.05% of the measured force, we have

$F = kq_1q_2/r^2;$

$(0.05 \times 10^{-2})(7 \times 10^{-7}\ \text{N}) = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)q^2/(0.10\ \text{m})^2$, which gives

$q = \boxed{2.0 \times 10^{-11}\ \text{C}}.$

$\boxed{23}$. (a) We consider two equal charges of magnitude 1 C separated by 1 cm. We call q_1 the magnitude in esu. If we equate the force in the two systems of units, we have

$F = q_1^2/r^2 = kq^2/r^2$;

$[q_1^2/(1\text{ cm})^2](10^{-5}\text{ N/dyne}) = (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1\text{ C})^2/(1 \times 10^{-2}\text{ m})^2$, which gives

$q_1 = \boxed{3 \times 10^9\text{ esu in 1 C}}$.

(b) The electron charge is

$e = (1.6 \times 10^{-19}\text{ C})(3 \times 10^9\text{ esu/C}) = \boxed{4.8 \times 10^{-10}\text{ esu}}$.

24. (a) The attractive Coulomb force provides the centripetal acceleration:

$F = mv^2/r = mr\omega^2$;

$k\,e^2/r^2 = mr\omega^2$, which we write as $ke^2 = mr^3(2\pi/T)^2$;

$(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})^2 = (9.11 \times 10^{-31}\text{ kg})r^3[2\pi/(24\text{ h})(3600\text{ s/h})]^2$,

which gives $r = \boxed{3.6 \times 10^3\text{ m}}$.

(b) For the hydrogen orbit, we have

$(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})^2 = (9.11 \times 10^{-31}\text{ kg})r^3[2\pi/(4 \times 10^{-16}\text{ s})]^2$,

which gives $r = \boxed{1.0 \times 10^{-10}\text{ m}}$.

25. The Coulomb force is

$F = kq_1q_2/r^2 = kq_1(q - q_1)/r^2 = (qq_1 - q_1^2)k/r^2$, with q_1 as the variable.

To find q_1 that maximizes the force, we set $dF/dq_1 = 0$:

$dF/dq_1 = (q - 2q_1)k/r^2 = 0$, which gives $\boxed{q_1/q = \tfrac{1}{2}}$.

This means that $\boxed{q_2/q = \tfrac{1}{2}}$, which we would expect from the symmetry of the force law.

26. The two particles repel each other. At the position of closest approach, we have

$F = kq_1q_2/r^2 = k(2e)(74e)/r^2$

$= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2)(74)(1.6 \times 10^{-19}\text{ C})^2/(6.0 \times 10^{-12}\text{ m})^2$

$= \boxed{9.5 \times 10^{-4}\text{ N}}$ repulsion.

27. (a) The opposite charges attract. We find the magnitude of the Coulomb force from

$F = ke^2/r^2 = (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})^2/(3 \times 10^{-10}\text{ m})^2$

$= \boxed{2.6 \times 10^{-9}\text{ N toward the proton (centripetal)}}$.

(b) The attractive Coulomb force provides the centripetal acceleration:

$F = mv^2/r$;

$(2.6 \times 10^{-9}\text{ N}) = (9.11 \times 10^{-31}\text{ kg})v^2/(3 \times 10^{-10}\text{ m})$, which gives

$v = \boxed{9.2 \times 10^5\text{ m/s}}$.

(c) We find the frequency from

$f = v/2\pi r = (9.2 \times 10^5\text{ m/s})/2\pi(3 \times 10^{-10}\text{ m}) = \boxed{4.9 \times 10^{14}\text{ Hz}}$.

(d) We find the spring constant from

$k = (2\pi f)^2 m = [2\pi(4.9 \times 10^{14}\text{ Hz})]^2(9.11 \times 10^{-31}\text{ kg}) = \boxed{8.6\text{ N/m}}$.

28. (a) The acceleration of each particle is caused by the same force:

$F = m_1a_1 = m_2a_2$, which gives

$m_2 = (a_1/a_2)m_1 = [(1.93\text{ m/s}^2)/(5.36\text{ m/s}^2)](31.3\text{ g}) = \boxed{11.3\text{ g}}$.

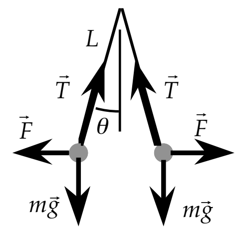
(b) Because the particles have equal charges, we have

$k\,q^2/r^2 = m_1a_1$;

$(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)q^2/(8.75 \times 10^{-2}\text{ m})^2 = (31.3 \times 10^{-3}\text{ kg})(1.93\text{ m/s}^2)$, which gives

$q = \boxed{2.27 \times 10^{-7}\text{ C}}$.

29. From the force diagram, we apply $\sum \vec{F} = 0$:
    horizontal: $T \sin \theta = F = kqq/r^2$;
    vertical: $T \cos \theta = mg$.
    If we divide the two equations, we get
    $\tan \theta = F/mg = kq^2/r^2 mg = kq^2/(2L \sin \theta)^2 mg$
    $\tan 10° = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2}) q^2/[2(0.20\ \mathrm{m}) \sin 10°]^2 (0.20 \times 10^{-3}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})$,
    which gives $q = \boxed{1.4 \times 10^{-8}\ \mathrm{C}}$.



30. (a) We find the magnitude of the electrical force from
    $F_E = kq^2/r^2$
    $= (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(8.5 \times 10^{15}\ \mathrm{C})^2/(3.8 \times 10^8\ \mathrm{m})^2 = \boxed{4.5 \times 10^{24}\ \mathrm{N}}$.
    (b) The ratio of forces is
    $F_E/F_g = (kq^2/r^2)/(GmM/r^2) = kq^2/GmM$
    $= (9 \times 10^9\ \mathrm{N \cdot m^2/C^2})(8.5 \times 10^{15}\ \mathrm{C})^2/(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(7.36 \times 10^{22}\ \mathrm{kg})(5.98 \times 10^{24}\ \mathrm{kg})$
    $= \boxed{2.2 \times 10^4}$.
    (c) The density of charge of the earth is
    $\rho = Q/V = (8.5 \times 10^{15}\ \mathrm{C})/(4/3)\pi(6.4 \times 10^6\ \mathrm{m})^3 = \boxed{7.7 \times 10^{-6}\ \mathrm{C/m^3}}$.
    (d) The density of protons to produce the charge density of part (c) is
    $\rho_p = \rho/e = (7.7 \times 10^{-6}\ \mathrm{C/m^3})/(1.6 \times 10^{-19}\ \mathrm{C/proton}) = \boxed{4.8 \times 10^{13}\ \mathrm{protons/m^3}}$.
    (e) The density of all protons in Earth is
    $\rho_p' = \frac{1}{2}\rho_M/m_p = \frac{1}{2}(5.52 \times 10^3\ \mathrm{kg/m^3})/(1.67 \times 10^{-27}\ \mathrm{kg/proton}) = \boxed{1.7 \times 10^{30}\ \mathrm{protons}}$

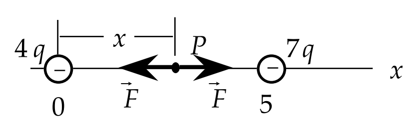
31. Because q_1 and q_2 attract each other, they must have opposite signs and their product will be negative. We can take this into account by giving the force a negative value:
    $F_{12} = kq_1q_2/r_{12}^2$;
    $-1.4 \times 10^{-2}\ \mathrm{N} = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2}) q_1q_2/(15.0 \times 10^{-2}\ \mathrm{m})^2$, which gives
    $q_1q_2 = -3.5 \times 10^{-14}\ \mathrm{C^2}$.
    Because q_2 and q_3 attract each other, they must have opposite signs and their product will be negative. We can take this into account by giving the force a negative value:
    $F_{23} = kq_2q_3/r_{23}^2$;
    $-3.8 \times 10^{-3}\ \mathrm{N} = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2}) q_2q_3/(20.0 \times 10^{-2}\ \mathrm{m})^2$, which gives
    $q_2q_3 = -1.7 \times 10^{-13}\ \mathrm{C^2}$.
    Because q_1 and q_3 repel each other, they must have the same sign and their product will be positive. We can take this into account by giving the force a positive value:
    $F_{13} = kq_1q_3/r_{13}^2$;
    $+5.2 \times 10^{-3}\ \mathrm{N} = (9 \times 10^9\ \mathrm{N \cdot m^2/C^2}) q_1q_3/(10.0 \times 10^{-2}\ \mathrm{m})^2$, which gives
    $q_1q_3 = +5.8 \times 10^{-14}\ \mathrm{C^2}$.
    We have three equations for three unknowns, q_1, q_2, and q_3. If we assume that q_1 is positive, when we combine the equations we get
    $q_1 = \boxed{+1.1 \times 10^{-7}\ \mathrm{C}}$,
    $q_2 = \boxed{-3.2 \times 10^{-7}\ \mathrm{C}}$,
    $q_3 = \boxed{+5.3 \times 10^{-7}\ \mathrm{C}}$.
    If we took q_1 to be negative, we would get the same magnitudes, with q_1 and q_3 negative and q_2 positive.

32. (a) In order to have a repulsion, Q must be negative. For forces with equal magnitudes, we have
$GMm/r^2 = kQe/r^2$, so we get
$Q = -GMm/ke$
$= -(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6 \times 10^{24}\ \text{kg})(0.9 \times 10^{-30}\ \text{kg})/(9 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})$
$= \boxed{-2.5 \times 10^{-7}\ \text{C}}$.

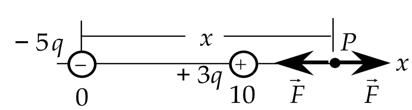
(b) The number of protons in the earth is
$N_p = \tfrac{1}{2}M/m_p$.
The discrepancy between the proton and the electron charges as a fraction of the electron charge is
$\Delta = (Q/N_p)/e = 2Qm_p/Me$
$= 2(2.5 \times 10^{-7}\ \text{C})(1.6 \times 10^{-27}\ \text{kg})/(6 \times 10^{24}\ \text{kg})(1.6 \times 10^{-19}\ \text{C})$
$= \boxed{8 \times 10^{-40}\ \text{of electron charge}}$.

33. The force on a mass m attracted to a fixed mass M is $F_g = GMm/r^2$. The potential energy of the mass m is
$U = -GMm/r$, with $U = 0$ when $r = \infty$.
The negative sign for U is due to the force being attractive; the potential energy decreases as the masses come closer.
The force on a charge q repelled by a fixed charge Q of the same sign is $F_E = kQq/r^2$. The potential energy of the charge q is
$U_E = +kQq/r$, with $U = 0$ when $r = \infty$.
The positive sign for U is due to the force being repulsive; the potential energy increases as the charges come closer.
If the electrical force is the only one present, we have energy conservation. If the moving point charge is aimed straight at the fixed charge, at the distance of closest approach the charge will momentarily come to rest. Thus we have
$K_i + U_i = K_f + U_f$;
$K_i + 0 = 0 + (kq_1q_2/r_f)$;
$1\ \text{J} = (9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10^{-6}\ \text{C})(10^{-4}\ \text{C})/r_f$, which gives
$r_f = \boxed{0.9\ \text{m}}$.

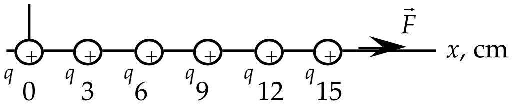
34. Because the two charges have the same sign, the charge Q must be between the two on the x-axis, where the two forces on Q will be in opposite directions. The net force will be zero when the two magnitudes are equal:
$k4qQ/r_1^2 = k7qQ/r_2^2$, or, when we cancel common factors,
$4/x^2 = 7/(5 - x)^2$, which gives $x = 2.15$, and -15.4.
The point is between the two charges at $\boxed{x = 2.15}$.



35. Because the two charges have opposite signs, the charge Q must be on the x-axis outside the two, where the two forces on Q will be in opposite directions. The net force will be zero when the two magnitudes are equal
$k5qQ/r_1^2 = k3qQ/r_2^2$, or, when we cancel common factors,
$5/x^2 = 3/(x - 10)^2$, which gives $x = 5.6$, and 44.4.
The point is outside the two charges at $\boxed{x = 44.4}$.
Compare with the answer to Problem 34.



36.



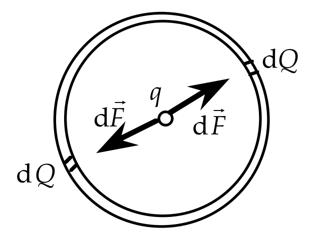
Because all of the charges have the same sign, the charge at $x = 25$ cm is repelled by all of the others. The net force will be toward $+x$, with a magnitude equal to the sum of the magnitudes of the individual forces:

$$F = \sum_{n=1}^{5} \frac{kqq}{r_i^2} = kqq\left[\frac{1}{(0.25\text{ m})^2} + \frac{1}{(0.20\text{ m})^2} + \frac{1}{(0.15\text{ m})^2} + \frac{1}{(0.10\text{ m})^2} + \frac{1}{(0.05\text{ m})^2}\right]$$

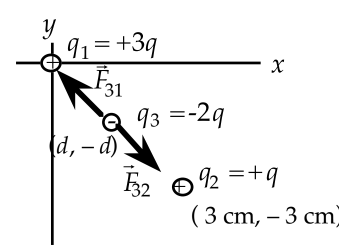
$$= \frac{(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.4 \times 10^{-7}\text{ C})^2}{(0.05\text{ m})^2}\left[\frac{1}{(5)^2} + \frac{1}{(4)^2} + \frac{1}{(3)^2} + \frac{1}{(2)^2} + \frac{1}{(1)^2}\right] = 0.30\text{ N}.$$

The net force is $\boxed{0.30\text{ N in the } +x\text{ -direction}}$.

$\boxed{37}$. The ring of charge Q can be thought of as an infinite number of differential charges spread uniformly on the ring. The positive charge q is attracted by one of the differential charges, which has a matching charge on the opposite side of the ring. The sum of the forces from the pair is zero, thus when all pairs are considered, the net force on q must be $\boxed{\text{zero}}$.



38. There will be two forces acting on the third charge. When the third charge is in equilibrium, the net force is zero, so the two forces must be in opposite directions. Because the sign of the third charge is opposite to the other two charges, it is attracted by the other two, so the third charge must be between the other two charges, which are separated by $3\sqrt{2}$ cm along the line $y = -x$. We place the third charge at ($+d$ cm, $-d$ cm), with $d < 3$ cm. The magnitudes of the two forces must be equal:



$F_{31} = F_{32}$;

$k\,q_3q_1/r_{31}^2 = kq_3q_2/r_{32}^2$, which reduces to

$q_1/r_{31}^2 = q_2/r_{32}^2$;

$q/(d\sqrt{2})^2 = 3q/[(3\sqrt{2}\text{ cm}) - (d\sqrt{2})]^2$, which gives $d = 1.10$ cm.

The third charge must be at $\boxed{(+1.10\text{ cm}, -1.10\text{ cm})}$.

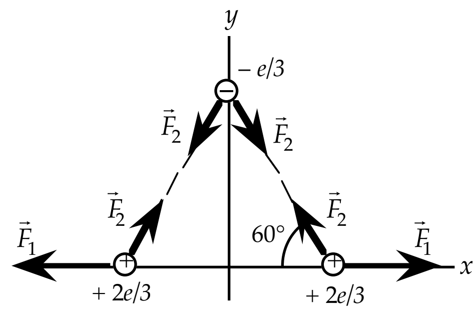
If the third charge is displaced slightly along the line joining the charges, the charge toward which it is moved will exert a larger attracting force, so the net force will be away from the equilibrium position. The equilibrium will be unstable.

If the third charge is displaced slightly away from the line joining the charges, both attracting forces will have a component back toward the line, so the net force will be toward the equilibrium position. The equilibrium will be stable.

39. The forces on each quark are shown in the diagram.
For the positive "up" quark on the right, we have

$\sum F_x = F_1 - F_2 \cos 60°$
$= (46 \text{ N}) - (23 \text{ N}) \cos 60° = 34 \text{ N};$

$\sum F_y = F_2 \sin 60°$
$= (23 \text{ N}) \sin 60° = +20 \text{ N}.$



When we combine these components, we get

$F_+ = \boxed{39 \text{ N } 30° \text{above the line joining the two "up" quarks}}$

(the x-axis).
From symmetry, the force on the left "up" quark will be 39 N 30°above the – x-axis.
For the negative "down" quark at the top, we have

$\sum F_x = F_2 \cos 60° - F_2 \cos 60° = 0;$

$\sum F_y = - F_2 \sin 60° - F_2 \sin 60°$
$= - 2(23 \text{ N}) \sin 60° = - 40 \text{ N}.$

The force on the "down" quark is

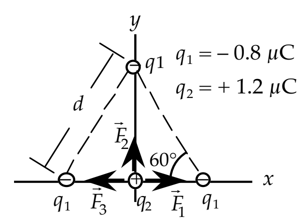
$F_- = \boxed{40 \text{ N toward the center of the line joining the two "up" quarks}}.$

Note that the sum of the three forces is zero, within the limitation of significant figures.

40. The three forces acting on the positive charge are shown in the diagram.
Their magnitudes are

$F_1 = F_3 = k\,|q_1|\,|q_2| / (d/2)^2$
$= (9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(0.6 \times 10^{-6} \text{ C})(1.5 \times 10^{-6} \text{ C}) / (9.0 \times 10^{-2} \text{ m})^2$
$= 1.76 \text{ N};$

$F_2 = k\,|q_1|\,|q_2| / (d \sin 60°)^2$
$= (9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(0.6 \times 10^{-6} \text{ C})(1.5 \times 10^{-6} \text{ C}) / (18 \times 10^{-2} \sin 60° \text{ m})^2$
$= 0.59 \text{ N}.$



From the symmetry of the forces, we have

$\vec{F}_1 + \vec{F}_3 = 0;$

$\vec{F} = \vec{F}_2 = (0.59 \text{ N})\,\hat{j}\,.$

The net force is $\boxed{0.59 \text{ N toward the opposite corner}}$.

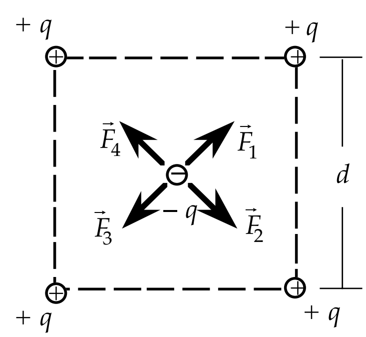
$\boxed{41}$. (a) From the symmetry of the charges and the distances, we have

$F_1 = F_2 = F_3 = F_4$, so

$\sum \vec{F} = 0$, the negative charge is in equilibrium.

(b) If the negative charge is moved slightly toward one of the positive charges, the attractive force toward that charge will increase, while the attractive force toward the opposite corner will decrease. The net force will be away from the equilibrium point, so it will be $\boxed{\text{unstable}}$.

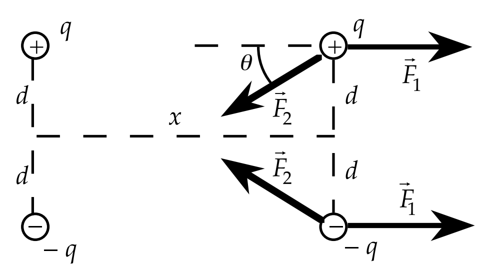
(c) If the negative charge is moved perpendicular to the plane a small distance, each of the four attractive forces will have a component pointing back toward the plane. The net force, the sum of these four forces, will be toward the equilibrium point, so it will be $\boxed{\text{stable}}$.



42. From the symmetry of the charge distribution, we know that the forces on the dipoles will have the same magnitude and opposite directions, so we consider one dipole. Each charge of the dipole will have two forces acting on it, as shown in the diagram. From the symmetry of the charges and distances, we see that the net force will be horizontal, and the force on the positive charge will be the same as that on the negative charge. The net force on the dipole is

$$F_{\text{net}} = 2\left(F_1 - F_2 \cos\theta\right) = 2\left\{\frac{kq^2}{x^2} - \frac{kq^2}{x^2 + (2d)^2}\,\frac{x}{\left[x^2 + (2d)^2\right]^{1/2}}\right\}$$

$$= \frac{2kq^2}{x^2}\left\{1 - \frac{1}{\left[1 + \left(4d^2/x^2\right)\right]^{3/2}}\right\} = \frac{2kq^2}{x^2}\left\{1 - \left[1 + \left(4d^2/x^2\right)\right]^{-3/2}\right\}.$$



When $d \ll x$, $[1 + (4d^2/x^2)]^{-3/2} \simeq 1 - \frac{3}{2}(4d^2/x^2)$, so we have

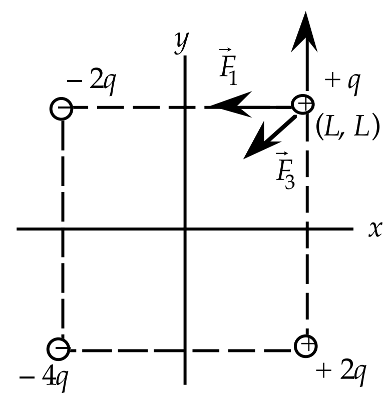
$$F_{\text{net}} \simeq \frac{2kq^2}{x^2}\left\{1 - \left[1 - \frac{3}{2}\left(\frac{4d^2}{x^2}\right)\right]\right\} = \frac{12kd^2q^2}{x^4}.$$

43. (a) The three forces acting on q are shown in the figure. Their magnitudes are

$$F_1 = F_2 = k2qq/(2L)^2 = \tfrac{1}{2}kq^2/L^2;$$
$$F_3 = k4qq/(2L\sqrt{2})^2 = \tfrac{1}{2}kq^2/L^2.$$

The net force acting on q is

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (-\tfrac{1}{2}kq^2/L^2)\,\hat{i} + (\tfrac{1}{2}kq^2/L^2)\,\hat{j} - \{[(\tfrac{1}{2}kq^2/L^2)\cos 45°]\,\hat{i} + [(\tfrac{1}{2}kq^2/L^2)\sin 45°]\,\hat{j}\}$$
$$= (\tfrac{1}{2}kq^2/L^2)\{[-(2+\sqrt{2})/2]\,\hat{i} + [(2-\sqrt{2})/2]\,\hat{j}\}$$
$$= \boxed{(\sqrt{3})kq^2/2L^2,\ 9.7° \text{ above the } -x\text{-axis}}.$$

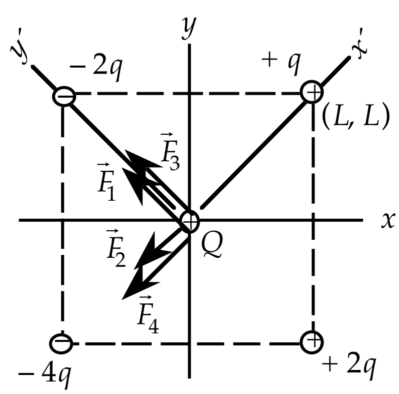


(b) The four forces acting on Q are shown in the figure. Their magnitudes are

$$F_1 = F_3 = k2qQ/(L\sqrt{2})^2 = kqQ/L^2;$$
$$F_2 = kqQ/(L\sqrt{2})^2 = kqQ/2L^2;$$
$$F_4 = k4qQ/(L\sqrt{2})^2 = 2kqQ/L^2.$$

To find the net force, we use a rotated $x'y'$-coordinate system, as shown on the diagram. Thus

$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$
$$= (kqQ/L^2)\,\hat{j}\,' - (kqQ/2L^2)\,\hat{i}\,' + (kqQ/L^2)\,\hat{j}\,' - (2kqQ/L^2)\,\hat{i}\,'$$
$$= (kqQ/L^2)[-2.5\,\hat{i}\,' + 2\,\hat{j}\,']$$
$$= 3.2kqQ/L^2,\ 38.7° \text{ above the } -x'\text{-axis, or } \boxed{3.2kqQ/L^2,\ 6.3° \text{ below the } -x\text{-axis}}.$$



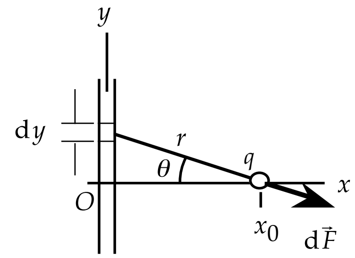
44. (a) Because the charge Q is symmetrically distributed about $y = 0$, the force on q will be along the x-axis toward positive x.

(b) Because Q is distributed uniformly, the linear charge density is $\lambda = Q/2L$, and the charge on a segment is
$dQ = \boxed{(Q/2L)\,dy}$.

(c) The force vector, shown in the figure, has magnitude
$dF = (kq/r^2)\,dQ = \boxed{(kqQ/2Lr^2)\,dy}$.

(d) From part (a), we know that we add (integrate) the x-components:

$$F_x = \int dF_x = \int_{-L}^{L} \frac{kqQ}{2Lr^2}\cos\theta\,dy = \int_{-L}^{L} \frac{kqQ}{2L}\frac{D}{\left(D^2+y^2\right)^{3/2}}\,dy.$$



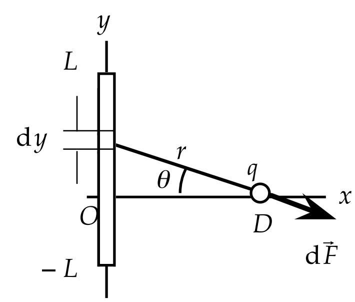
(e) The result of the integration is

$$F_x = \frac{kqQD}{2L}\left.\frac{y}{D^2\left(D^2+y^2\right)^{1/2}}\right|_{-L}^{L} = \frac{kqQ}{2LD}\left[\frac{L}{D^2\left(D^2+L^2\right)^{1/2}} - \frac{-L}{D^2\left(D^2+L^2\right)^{1/2}}\right]$$

$$= \frac{kqQ}{D\left(D^2+L^2\right)^{1/2}}.$$

45. Because the line charge is symmetrically distributed about $y = 0$, the force on q will be along the x-axis toward positive x.
The charge on the segment dy is
$dQ = \lambda\,dy$.
The force vector, shown in the figure, has magnitude
$dF = (kq/r^2)\,dQ = (kq\lambda/r^2)\,dy$.
To find the force, we add (integrate) the x-components:

$$F = \int dF_x = \int_{-\infty}^{\infty} \frac{kq\lambda}{r^2}\cos\theta\,dy.$$



From the figure, we see that $r = x_0/\cos\theta$, and $y = x_0\tan\theta$.
We change variable to θ, with
$dy = x_0\sec^2\theta\,d\theta = (x_0/\cos^2\theta)\,d\theta$:

$$F = \int_{-\pi/2}^{\pi/2} \frac{kq\lambda}{\left(x_0/\cos\theta\right)^2}\left(\cos\theta\right)\left(x_0/\cos^2\theta\right)d\theta = \int_{-\pi/2}^{\pi/2} \frac{kq\lambda}{x_0}\cos\theta\,d\theta$$

$$= \frac{kq\lambda}{x_0}\left(\sin\theta\right)\Big|_{-\pi/2}^{\pi/2} = \frac{2kq\lambda}{x_0}.$$

$\vec{F} = (2kq\lambda/x_0)\,\hat{i}$.

46. We use the figure from Problem 45, with the line charge from $y = 0$ to $y = +\infty$. The force on q will have both an x-component and a y-component. We find each component by integrating, using the same change of variable that we used in Problem 43:

$$F_x = \int dF_x = \int_0^{\infty} \frac{kq\lambda}{r^2} \cos\theta \, dy$$

$$= \int_0^{\pi/2} \frac{kq\lambda}{(x_0/\cos\theta)^2} (\cos\theta)(x_0/\cos^2\theta)\, d\theta = \int_0^{\pi/2} \frac{kq\lambda}{x_0} \cos\theta \, d\theta$$

$$= \frac{kq\lambda}{x_0} (\sin\theta)\Big|_0^{\pi/2} = \frac{kq\lambda}{x_0};$$

$$F_y = \int dF_y = -\int_0^{\infty} \frac{kq\lambda}{r^2} \sin\theta \, dy.$$

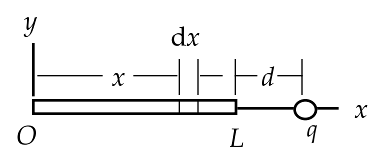
$$= -\int_0^{\pi/2} \frac{kq\lambda}{(x_0/\cos\theta)^2} (\sin\theta)(x_0/\cos^2\theta)\, d\theta = -\int_0^{\pi/2} \frac{kq\lambda}{x_0} \sin\theta \, d\theta$$

$$= -\frac{kq\lambda}{x_0} (-\cos\theta)\Big|_0^{\pi/2} = -\frac{kq\lambda}{x_0}.$$

The force on q is

$\vec{F} = (kq\lambda/x_0)\,\hat{i} - (kq\lambda/x_0)\,\hat{j}$, or $\boxed{(kq\lambda/x_0)\sqrt{2}\ \ 45° \text{ below the } x\text{-axis}}$.

$\boxed{47}$. We align the rod along the x-axis with one end at the origin, as shown in the figure. The linear charge density is $\lambda = Q/L$, so the charge on the element dx is $dQ = (Q/L)\,dx$. All elements of the rod produce a force in the $+x$-direction. The total force is



$$\vec{F} = \int \hat{i}\, dF_x = \int_0^L \frac{kq\lambda}{r^2}\hat{i}\, dx = \frac{kqQ}{L}\hat{i}\int_0^L \frac{dx}{(L - x + d)^2}$$

$$= \frac{kqQ}{L}\hat{i}\left(\frac{1}{L - x + d}\right)\Bigg|_0^L = \frac{kqQ}{L}\hat{i}\left(\frac{1}{d} - \frac{1}{L + d}\right) = \frac{kqQ}{d(L + d)}\hat{i}.$$

The force on q is $\boxed{kqQ/d(L + d) \text{ away from the rod}}$.

48. In Example 21-10 we found the electric force exerted on a point charge Q located on the axis of a uniformly charged ring of charge q and radius R to be

$F = kqQL/(R^2 + L^2)^{3/2}$,

where L is the distance between Q and the center of the ring. In our case there are two rings, each exerting a force on Q. Since these two forces are opposite in direction and we want the net force to be zero, we need to place Q where the two forces have the same magnitude. Let the distance between the first ring and Q be L_1, etc, then we have

$F_1 = F_2$;

$kqQL_1/(R_1^2 + L_1^2)^{3/2} = kqQL_2/(R_2^2 + L_2^2)^{3/2}$.

Also, $L_1 + L_2 = 100$ cm. Plug in $R_1 = 25$ cm and $R_2 = 40$ cm and solve for L_1: $L_1 = \boxed{1.3 \text{ cm}}$, i.e., the charge should be placed 1.3 cm from the center of the ring with a radius of 25 cm (and 98.3 cm from the other).

49. Let the charge on each ring of radius R be q and the center-to-center separation between the two rings be $2L$. Draw an x-axis from the center of one ring (at $x = -L$) to that of the other (where $x = +L$). By symmetry the net force of both rings on a point charge Q is zero at $x = 0$, midway between the two rings. To examine the stability of the equilibrium position at $x = 0$, imagine moving the point charge to a new location x, away form (yet very close to) the equilibrium: $|x| << L$. The point charge is now a distance $(L + x)$ from the one ring and $(L - x)$ from the other. The net force exerted on the point charge is now

$$F = kqQ(L + x)/[R^2 + (L + x)^2]^{3/2} - kqQ(L - x)/[R^2 + (L - x)^2]^{3/2}.$$

For $|x| << L$ we may use the approximation

$$\begin{aligned}(L + x)[R^2 + (L + x)^2]^{-3/2} &\approx (L + x)(R^2 + L^2 + 2Lx)^{-3/2}\\ &= (L + x)(R^2 + L^2)^{-3/2}\,[1 + 2Lx/(R^2 + L^2)]^{-3/2}\\ &\approx (L + x)(R^2 + L^2)^{-3/2}\,[1 + (-3/2)2Lx/(R^2 + L^2)]\\ &\approx L/(R^2 + L^2)^{3/2} + [(R^2 - 2L^2)/(R^2 + L^2)^{5/2}]x \quad \text{(up to the first power in } x\text{)}\end{aligned}$$

and

$$[R^2 + (L - x)^2]^{-3/2} \approx L/(R^2 + L^2)^{3/2} - [(R^2 - 2L^2)/(R^2 + L^2)^{5/2}]x.$$

so

$$F \approx [2kqQ(R^2 - 2L^2)/(R^2 + L^2)^{5/2}]x = Cx.$$

If $C > 0$, then $F(x)$ has the same sign as x. Thus if Q moves toward one of the two rings the net force on it tends to push it further toward that ring — the equilibrium is unstable. Similarly, if $C < 0$ then $F(x)$ and x have opposite signs, and the net force always tends to push the charge back to $x = 0$. The equilibrium is stable. (Just think of the restoring force of a spring, $F = -kx$, where $k > 0$.)
Thus the stability of the equilibrium depends on the sign of the expression

$$qQ(R^2 - 2L^2).$$

$\boxed{\text{If } q \text{ and } Q \text{ have the same sign, then the equilibrium is stable if } R > \sqrt{2}L\text{, and unstable if } R < \sqrt{2}L.}$

$\boxed{\text{If } q \text{ and } Q \text{ have opposite signs, then the equilibrium is stable if } R < \sqrt{2}L\text{, and unstable if } R > \sqrt{2}L.}$

50. We pair an element of the ring dQ with the element diametrically opposite. The two forces exerted on q at the center will have the same magnitude but opposite directions. Their sum will be zero, and thus, for all of the elements of the ring, we have

$$\vec{F} = 0.$$

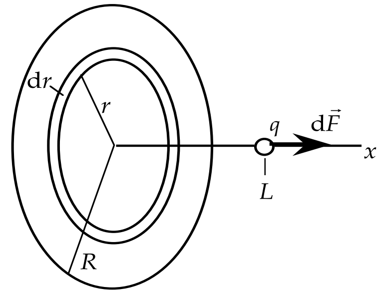
We assume that q and Q have the same sign. If q is moved in the xy-plane, the dQ toward which it moves will exert a greater repulsion, while the dQ on the opposite side will exert a smaller repulsion. The net force will be toward the center of the ring; the equilibrium is stable. If q and Q have opposite signs, the forces become attractive. The net force will be away from the center of the ring; the equilibrium is unstable.
The Coulomb force is an inverse-square force, like the gravitational force. A mass anywhere inside a uniform spherical shell of mass will experience no gravitational force. A charge inside a uniformly charged spherical shell will experience no electrical force.

51. If we consider the plate to be a series of concentric rings, each ring will produce a force away from the plate, as shown in the figure. We choose a representative ring of radius r and thickness dr. The area charge density of the plate is

$\sigma = Q/\pi R^2$, so the charge on the ring is

$dq = \sigma 2\pi r\, dr = (2Q/R^2)r\, dr$.

We use the result for a ring from Example 21–10 to find the total force by summing (integrating) the forces from all of the rings:



$$\vec{F} = \int d\vec{F} = \int_0^R \frac{2kqQL}{R^2} \frac{r\, dr}{\left(r^2 + L^2\right)^{3/2}} \hat{i} = \frac{2kqQL}{R^2} \hat{i} \left[\frac{-1}{\left(r^2 + L^2\right)^{1/2}}\right]\Bigg|_0^R$$

$$= \frac{2kqQL}{R^2}\left[\frac{-1}{\left(R^2 + L^2\right)^{1/2}} - \frac{-1}{L}\right]\hat{i} = \frac{2kqQ}{R^2}\left[1 - \frac{L}{\left(R^2 + L^2\right)^{1/2}}\right]\hat{i}.$$

For the given data, we get

$$\vec{F} = \frac{2\left(9 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(0.65 \times 10^{-6}\ \text{C}\right)\left(1.6 \times 10^{-6}\ \text{C}\right)}{\left(8 \times 10^{-2}\ \text{m}\right)^2} \times$$

$$\left\{1 - \frac{5\ \text{cm}}{\left[\left(8\ \text{cm}\right)^2 + \left(5\ \text{cm}\right)^2\right]^{1/2}}\right\}\vec{i}$$

$= 1.4$ N away from the center.

52. We can consider the plane sheet as a plate with an infinite radius. The analysis of Problem 51 can be used:

$$F = \int dF = \int_0^\infty kq\sigma L \frac{2\pi r\, dr}{\left(r^2 + L^2\right)^{3/2}}$$

$$= 2\pi kq\sigma L\left[\frac{-1}{\left(r^2 + L^2\right)^{1/2}}\right]\Bigg|_0^\infty = 2\pi kq\sigma L\left(\frac{1}{L}\right) = 2\pi kq\sigma$$

$= \dfrac{q\sigma}{2\varepsilon_0}$ away from the plane sheet.

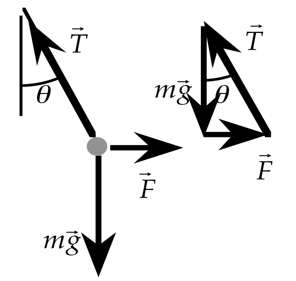
53. Because the plates attract each other, equal and opposite forces are required to separate them. The two plates are so close that we can say that, to any small element of charge on one plate, the other plate will appear infinite. This neglects small effects at the edge. The field of the plate is uniform due to the charge density $\sigma = Q/A$. From the result of Problem 52, the force on a small segment of charge ΔQ due to the charge on the other plate is

$\Delta F = \Delta Q\ (Q/A)/2\varepsilon_0$.

When we add the forces on all of the charge elements, we have

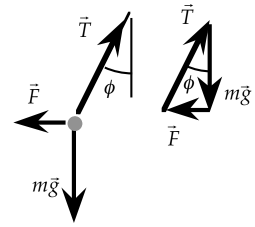
$F = \sum \Delta Q\ (Q/2\varepsilon_0 A) = Q^2/2\varepsilon_0 A$;

$0.1\ \text{N} = /2(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.05\ \text{m}^2)$, which gives $Q = \boxed{3.0 \times 10^{-7}\ \text{C}}$.



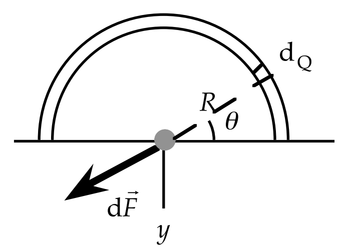
54. From the result of Problem 52, we know that the force exerted by the sheet of charge will be perpendicular to the sheet.
   (a) A positive charge is repelled by the sheet. The cork ball will hang at an angle where $\sum \vec{F} = 0$. From the diagram of the vector sum of the forces, we have

$$\tan\theta = F/mg = (q\sigma/2\varepsilon_0)/mg = q\sigma/2\varepsilon_0 mg$$
$$= (0.8\times10^{-8}\ \mathrm{C})(1.2\times10^{-6}\ \mathrm{C/m^2})/[2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(8\times10^{-3}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})]$$
$$= 0.0069, \text{ so } \theta = \boxed{0.4^\circ \text{ from the vertical away from the sheet}}.$$



(b) A negative charge is attracted by the sheet. We find the angle from

$$\tan\phi = q\sigma/2\varepsilon_0 mg$$
$$= (3\times10^{-8}\ \mathrm{C})(2\times10^{-6}\ \mathrm{C/m^2})/[2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})(2\times10^{-3}\ \mathrm{kg})(9.8\ \mathrm{m/s^2})]$$
$$= 0.026, \text{ so } \phi = \boxed{1.5^\circ \text{ from the vertical toward the sheet}}.$$



55. We place the wire in a vertical plane, as shown. From the symmetry of the charge distribution, we know that the force on q will be down. The linear charge density of the wire is $\lambda = Q/\pi R$. We use an element $\mathrm{d}Q = \lambda R\ \mathrm{d}\theta$ at an angle θ from the horizontal. We find the net force by summing (integrating) the vertical components:

$$F = \int \mathrm{d}F_y = \int_0^\pi \frac{kq}{R^2}\sin\theta\ \mathrm{d}Q = \int_0^\pi \frac{kqQ}{\pi R^3}(\sin\theta)\ R\ \mathrm{d}\theta$$
$$= \frac{kqQ}{\pi R^2}(-\cos\theta)\Big|_0^\pi = \frac{2kqQ}{\pi R^2}.$$

From the given data, we get

$$F = 2(9\times10^9\ \mathrm{N\cdot m^2/C^2})(0.30\times10^{-6}\ \mathrm{C})(0.75\times10^{-6}\ \mathrm{C})/\pi(0.050\ \mathrm{m})^2 = \boxed{0.52\ \mathrm{N}}.$$

56. (a) All of the forces from the charges will lie along the line of the charges, with those from the positive charges to the right and those from the negative charges to the left. We allow for the possibility that the charges are not the same. We label a representative charge with m, where m goes from 0 to n. The distance from the mth charge to Q is $D - md$, and we can take the direction of the force into account by using a factor of $(-1)^m$. Thus we have

$$F = kQ\sum_{m=0}^{n}\frac{q_m(-1)^m}{(D-md)^2}. \qquad \text{When } q_m = q, \text{ we have } F = kQq\sum_{m=0}^{n}\frac{(-1)^m}{(D-md)^2}.$$

   (b) Because $D >> md$, we can use the approximation $(D - md)^{-2} \simeq D^{-2}[1 + (2md/D)]$:

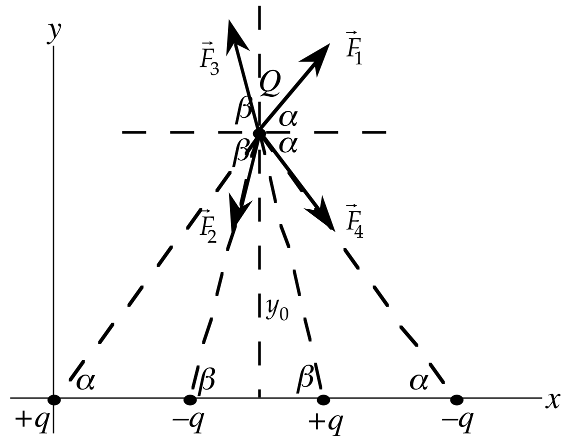
$$F = kQ\sum_{m=0}^{n}\frac{q_m(-1)^m}{D^2}\left(1+\frac{2md}{D}\right) = \frac{kQ}{D^2}\sum_{m=0}^{n} q_m(-1)^m + \frac{2kQd}{D^3}\sum_{m=0}^{n} mq_m(-1)^m.$$

When $q_m = q$, we have

$$F = \frac{kQq}{D^2}\sum_{m=0}^{n}(-1)^m + \frac{2kQqd}{D^3}\sum_{m=0}^{n} m(-1)^m, \text{ where we have different results for } n \text{ being odd or even:}$$

$$F_{\mathrm{odd}} = 0 - \frac{2kQqd}{D^3}\left(\frac{n+1}{2}\right) = -\frac{(n+1)kQqd}{D^3}, \quad \text{and} \quad F_{\mathrm{even}} = +\frac{kQq}{D^2} + \frac{2kQqd}{D^3}\left(\frac{n}{2}\right) = +\frac{kQq}{D^2} + \frac{nkQqd}{D^3}.$$

57. By symmetry the net force, $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$, is parallel to the x-axis, as $\vec{F}_1 + \vec{F}_4$ is along the positive x-direction and $\vec{F}_2 + \vec{F}_3$ is along the negative x-direction. So we only need to consider the x-component of each force. We have



$F_{1x} = F_{4x} = F_1 \cos\alpha$
$= \{kq(3q)/[(1.5\text{ cm})^2 + y_0^2]\}\{1.5\text{ cm}/[(1.5\text{ cm})^2 + y_0^2]^{1/2}\}$
$= (4.5\text{ cm})kq^2/[(1.5\text{ cm})^2 + y_0^2]^{3/2}$, and
$F_{2x} = F_{3x} = -F_2 \cos\beta$
$= -\{kq(3q)/[(0.5\text{ cm})^2 + y_0^2]\}\{0.5\text{ cm}/[(0.5\text{ cm})^2 + y_0^2]^{1/2}\}$
$= -(1.5\text{ cm})kq^2/[(0.5\text{ cm})^2 + y_0^2]^{3/2}$.

Thus

$$\vec{F} = (F_{1x} + F_{2x} + F_{3x} + F_{4x})\hat{i} = (3.0\text{ cm})kq^2\{3/[(1.5\text{ cm})^2 + y_0^2]^{3/2} - 1/[(0.5\text{ cm})^2 + y_0^2]^{3/2}\}\hat{i}.$$

58. The charge is uniformly distributed over the entire sphere, with $\rho = e/V = e/(4\pi R^3/3)$. The portion of the charge that is contained in the spherical region of radius r is then $q = \rho\,(4\pi r^3/3) = er^3/R^3$. According to the textbook the net force exerted by the charged sphere on the negative point charge is then

$F = -keq/r^2 = -e^2r/R^3$.

Set this to equal to ma, with $a = \mathrm{d}^2r/\mathrm{d}t^2$ the acceleration of the point charge of mass m:

$-e^2r/R^3 = m\,\mathrm{d}^2r/\mathrm{d}t^2$, or
$m\,\mathrm{d}^2r/\mathrm{d}t^2 = -(e^2/R^3)r$.

This equation is analogous to the standard equation for a spring-mass system, namely, $m\,\mathrm{d}^2x/\mathrm{d}t^2 = -kx$, with x replaced by r and k by ke^2/R^3. Thus the solution to our equation also yields a simple-harmonic motion, equivalent to that with an effective spring constant of $k_{\text{eff}} = ke^2/R^3$. The frequency of the oscillation is

$f = \omega/2\pi = (1/2\pi)(k_{\text{eff}}/m)^{1/2} = \boxed{(1/2\pi)(ke^2/R^3m)^{1/2}}$.

59. We choose a differential charge element of one of the plates as $\mathrm{d}q = \sigma\,\mathrm{d}A$, which is a point charge. We find the force on $\mathrm{d}q$ exerted by the other plate from Problem 52:

$\mathrm{d}F = \mathrm{d}q\,(\sigma/2\varepsilon_0) = \sigma\,\mathrm{d}A(\sigma/2\varepsilon_0)$.

The force per unit area is

$\mathrm{d}F/\mathrm{d}A = \sigma^2/2\varepsilon_0 = (10^{-5}\text{ C/m}^2)^2/2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = \boxed{5.7\text{ N/m}^2}$.

The force per unit area is independent of the separation of the plates. If the distance is doubled, the force per unit area is $\boxed{5.7\text{ N/m}^2}$.

60. The surface area of the cone is

$A = \pi R(h^2 + R^2)^{1/2}$.

If the charge Q is uniformly distributed over its surface then the surface charge density is

$\sigma = Q/A = \boxed{Q/[\pi R(h^2 + R^2)^{1/2}]}$.

61. The surface charge density of Earth is

$\sigma = Q/A = Q/4\pi R^2$.

The electric field just outside its surface due to this charge density is $E = \sigma/2\varepsilon_0$, which exerts an electrostatic force of

$F_E = qE = q(\sigma/2\varepsilon_0) = q(Q/4\pi R^2)/2\varepsilon_0$

on a charge q placed near Earth's surface. For mechanical equilibrium for the charge of mass m, set

$F_E = F_g$; $q(Q/4\pi R^2)/2\varepsilon_0 = mg$; which gives
$q = 8\pi\varepsilon_0 R^2mg/Q$
$= 8\pi\,(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(6.37\times10^6\text{ m})^2)(10\times10^{-3}\text{ kg})(9.8\text{ m/s}^2)/(6\times10^5\text{ C}) = \boxed{1.5\times10^{-3}\text{ C}}$.

62. Because 1 mm is very small compared to the dimensions of the plate, we can treat the plate as an infinite plate with density $\sigma = Q/L^2$ and use the result of Problem 52. The upward electrical force is balanced by the downward force of gravity:

    $F_E = mg$, or $q\sigma/2\varepsilon_0 = mg$;

    $(0.8 \times 10^{-6}\text{ C})Q/[2(8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.60\text{ m})^2] = (1.5 \times 10^{-3}\text{ kg})(9.8\text{ m/s}^2)$; so

    $Q = \boxed{1.2 \times 10^{-7}\text{ C}}$.

    If $d = 2$ mm, the plate would still appear to be an infinite plate. The electrical force would not depend on distance, so Q remains 1.2×10^{-7} C.

    If $d = 1$ m, the plate would no longer appear to be infinite. The inverse-square dependence of the electrical force means that Q would have to be larger to exert the same magnitude force. For large distances, the plate would appear to be a point charge.

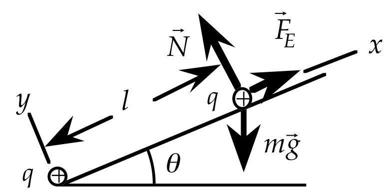
63. In the equilibrium position, the net force is zero. From the diagram,

    $\sum F_x = F_E - mg \sin\theta = 0$;

    $kqq/\ell^2 = mg \sin\theta$;

    $(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2 \times 10^{-8}\text{ C})^2/(0.08\text{ m})^2 = (0.5 \times 10^{-3}\text{ kg})(9.8\text{ m/s}^2) \sin\theta$, which gives

    $\sin\theta = 0.115$, $\theta = \boxed{6.6°}$.



64. There is a Coulomb force of repulsion between two like point charges:

    $F = (1/4\pi\varepsilon_0)qq/d^2$

    $= (9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\text{ C})(1.6 \times 10^{-19}\text{ C})/(8 \times 10^{-15}\text{ m})^2$

    $= \boxed{3.6\text{ N repulsion}}$.

$\boxed{65}$. (a) The attractive Coulomb force provides the centripetal acceleration:

    $F = (1/4\pi\varepsilon_0)(e^2/R^2) = mv^2/R$, which gives $v = \boxed{(e^2/4\pi\varepsilon_0 mR)^{1/2}}$.

(b) The magnitude of the angular momentum is

    $L = mvR = \boxed{(e^2 mR/4\pi\varepsilon_0)^{1/2}}$.

(c) We use the result of part (a):

    $v = (e^2 v/4\pi\varepsilon_0 L)^{1/2}$, which gives $v = \boxed{e^2/4\pi\varepsilon_0 L}$.

(d) We use the result of part (b):

    $R = L/mv = \boxed{4\pi\varepsilon_0 L^2/me^2}$.

(e) The time to go around the circle is the period:

    $\tau = 2\pi R/v = 2\pi(4\pi\varepsilon_0 L^2/me^2)/(e^2/4\pi\varepsilon_0 L) = \boxed{32\pi^3\varepsilon_0{}^2 L^3/me^4}$.

(f) We are given $L = 1.05 \times 10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}$. For the others, we have

    $v = (9 \times 10^9\text{ m/s}^2)(1.6 \times 10^{-19}\text{ C})^2/(1.05 \times 10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}) = \boxed{2.2 \times 10^6\text{ m/s}}$.

    $R = [1/(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)](1.05 \times 10^{-34}\text{ kg}\cdot\text{m}^2/\text{s})^2/(9.11 \times 10^{-31}\text{ kg})(1.6 \times 10^{-19}\text{ C})^2 = \boxed{5.3 \times 10^{-11}\text{ m}}$.

    $\tau = [2\pi/(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)^2](1.05 \times 10^{-34}\text{ kg}\cdot\text{m}^2/\text{s})^3/(9.11 \times 10^{-31}\text{ kg})(1.6 \times 10^{-19}\text{ C})^4 = \boxed{1.5 \times 10^{-16}\text{ s}}$.

66. The electrical force will be the repulsive force between the excess positive charge on the Sun and Earth. In each case, this will be the number of protons times δe. Set

    $F_E = F_g$

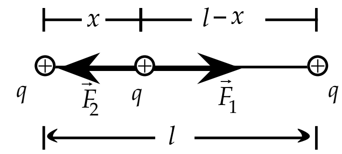
    $(1/4\pi\varepsilon_0)(\Delta q_{\text{sun}}\Delta q_{\text{earth}}/R^2) = (1/4\pi\varepsilon_0)(N_{\text{sun}}\,\delta e\, N_{\text{earth}}\,\delta e/R^2) = GM_{\text{sun}}M_{\text{earth}}/R^2$;

    $(9 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.25 \times 10^{57})\,\delta(1.15 \times 10^{44})\,\delta(1.6 \times 10^{-19}\text{ C})^2 =$

    $(6.7 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(2 \times 10^{30}\text{ kg})(6 \times 10^{24}\text{ kg})$, which gives

    $\delta = \boxed{5 \times 10^{-15}}$.

67. (a) The middle charge is repelled by each of the other charges. The net force is



$F_{\text{net}} = F_1 - F_2$;

$F_{\text{net}} = kq^2[1/x^2 - 1/(\ell - x)^2]$

$= \boxed{kq^2[\ell(\ell - 2x)/x^2(\ell - x)^2]\text{, away from the closer charge}}$.

For the net force to be zero, we have

$\ell - 2x = 0$, or $x = \ell/2$, as expected from symmetry.

(b) We call the displacement from equilibrium $\Delta x = x - (\ell/2)$. When we substitute this into the expression for the net force, we get

$F_{\text{net}} = kq^2[\ell(-2\,\Delta x)/(\ell/2 + \Delta x)^2(\ell/2 - \Delta x)^2]$, or

$\vec{F}_{\text{net}} = \boxed{-2kq^2\ell\,\Delta\vec{x}\,/[(\ell/2)^2 - (\Delta x)^2]^2}$.

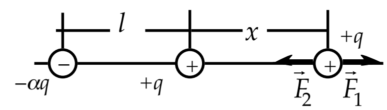
(c) When $\Delta x << \ell$, we can drop the Δx^2 term in the denominator:

$\vec{F}_{\text{net}} = -kq^2(32\,\Delta x/\ell^3)\,\hat{i}$, which has the form of a restoring spring force.

The oscillation frequency for small displacements is

$f = (1/2\pi)(k_{\text{eff}}/m)^{1/2} = \boxed{(1/2\pi)(32kq^2/\ell^3 m)^{1/2}}$.

68. (a) The net force will be



$$F_{\text{net}} = F_1 - F_2 = \frac{kq^2}{x^2} - \frac{k\alpha q^2}{(x+\ell)^2} = kq^2\frac{(x+\ell)^2 - \alpha x^2}{x^2(x+\ell)^2}.$$

(b) Because the charges have opposite signs, the moving charge must be outside of the two charges where the two forces will be in opposite directions and farther from the larger negative charge. The net force will be zero when

$(x + \ell)^2 - \alpha x^2 = 0$, or $(\alpha - 1)x^2 - 2\ell x - \ell^2 = 0$.

The positive solution to this quadratic equation is

$x = x_0 = \boxed{\ell[(1 + \alpha^{1/2})/(\alpha - 1)]}$.

The negative solution corresponds to a position between the two charges, where the equal magnitude forces will be in the same direction.

(c) If $\alpha = 40$, the equilibrium position is

$x_0 = \ell[(1 + \alpha^{1/2})/(\alpha - 1)] = \ell[(1 + 40^{1/2})/(40 - 1)] \approx 0.1878\ell$. To find the restoring force, we consider a small displacement Δx from the equilibrium position: $x = x_0 + \Delta x$, with $|\Delta x| << \ell$. The net force is

$F(x) \approx F(x_0) + (\mathrm{d}F/\mathrm{d}x)\Delta x = 0 + \{\mathrm{d}[kq^2/x^2 - \alpha kq^2/(x + \ell)^2]/\mathrm{d}x\}\,\Delta x$

$= 2kq^2[-1/x^3 + \alpha/(x + \ell)^3]\,\Delta x$

$\approx 2kq^2[-1/(0.1878\ell)^3 + 40/(0.1878\ell + \ell)^3]\,\Delta x$

$\approx -(254kq^2/\ell^3)\Delta x$

$= -k_{\text{effective}}\,\Delta x$.

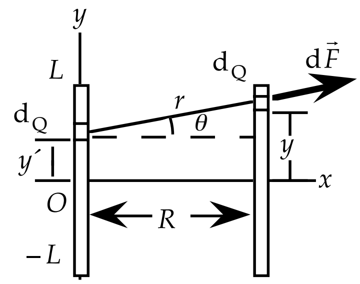
Thus the effective force constant is $254kq^2/\ell^3$, and the frequency of oscillations is

$f = (1/2\pi)(k_{\text{effective}}/m)^{1/2} = \boxed{(1/2\pi)(254kq^2/\ell^3 m)^{1/2}}$.

69. From the analogy with the gravitational force, we know that if we replace distribution 2 with a point charge q_2, the force exerted on q_2 by distribution 1 is the same as if distribution 1 were a point charge. From Newton's third law, the force on distribution 1 by q_2 is the reaction to the force on q_2, thus distribution 1 can be treated as a point charge when there is an external point charge. Similarly, if we replace distribution 1 with a point charge q_1, the force exerted on q_1 by distribution 2 is the same as if distribution 2 were a point charge. From Newton's third law, the force on distribution 2 by q_1 is the reaction to the force on q_1, thus distribution 2 can be treated as a point charge when there is an external point charge. Thus we can simultaneously treat both spherically symmetric distributions as point charges to find the force between them.

70. We find the force between the two rods by choosing a differential element for each rod, as shown in the diagram. The charge density of each rod is $\lambda = Q/2L$, so we have $dQ = (Q/2L)\,dy$ and $dQ' = (Q/2L)\,dy'$. These two elements are equivalent to two point charges, so the force between them is

$$dF = \frac{k\,dQ\,dQ'}{r^2} \text{ repulsion.}$$



From symmetry, the force between the two rods must be perpendicular to the rods. We need to add (integrate) only the x-component

$$F = \int dF_x = \iint \frac{k\,dQ\,dQ'}{r^2}\cos\theta$$

$$= \int_{-L}^{L}\int_{-L}^{L} k\left(\frac{Q}{2L}\right)^2 \frac{dy\,dy'}{r^2}\left(\frac{R}{r}\right) = \frac{kQ^2R}{4L^2}\int_{-L}^{L}\int_{-L}^{L}\frac{dy\,dy'}{\left[(y-y')^2+R^2\right]^{3/2}} \text{ repulsion.}$$

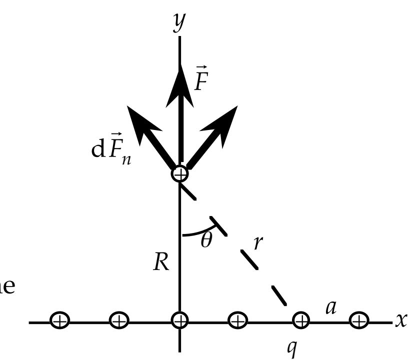
If $R >> L$, to each rod the other rod will be equivalent to a point charge, so $\boxed{F = kQ^2/R^2 \text{ repulsion}}$.

71. (a) If we consider a pair of charges equidistant from $x = 0$, we see that the x-component of the net force from the pair is zero. Thus the total force from the line of charges will be in the y-direction. We need to add only the y-components of the forces. This component from the nth charge is

$$F_{ny} = (kqQ/r^2)\cos\theta = (kqQ/r^2)(R/r)$$
$$= kqQR/r^3 = kqQR/[(na)^2 + R^2]^{3/2}.$$

The total force from all charges is

$$\vec{F} = \sum_{n=-\infty}^{\infty} F_{ny}\hat{j} = kqQR\sum_{n=-\infty}\left\{\frac{1}{\left[(na)^2+R^2\right]^{3/2}}\right\}\hat{j}.$$



(b) When $a \to 0$ and $q \to 0$ such that $q/a \to \lambda$, the distribution becomes a line charge. Each charge becomes dq; the location of the nth charge, na, becomes x; and the separation of charges, a, becomes dx. The summation becomes an integral:

$$\vec{F} = k\left(\frac{q}{a}\right)QR\sum_{n=-\infty}^{\infty}\left\{\frac{a}{\left[(na)^2+R^2\right]^{3/2}}\right\}\hat{j};$$

$$\vec{F} = k\lambda QR\int_{-\infty}^{\infty}\frac{dx}{\left(x^2+R^2\right)^{3/2}}\hat{j}.$$

We scale the integral by making the substitutions $x = uR$ and $dx = R\,du$:

$$\vec{F} = k\lambda QR\int_{-\infty}^{\infty}\frac{R\,du}{\left[(uR)^2+R^2\right]^{3/2}}\hat{j} = \frac{k\lambda Q}{R}\int_{-\infty}^{\infty}\frac{du}{\left(u^2+1\right)^{3/2}}\hat{j}.$$

Since u is dimensionless, so is the last integral. From the factor in front we see that $\vec{F}$ varies as $1/R$.